

DISSERTATIO PHYSICA,

DE

MUTATIONIBUS VOLUMINIS AQUÆ
DESTILLATÆ INTRA TEMPERATURAM
CONGELATIONIS ET VICESIMI GRADUS IN
THERMOMETRO CENTESIMALI.

QUAM

CONSENTIENTE AMPL. FAC. PHIL. REG. ACAD. ABOËNSIS,

PRÆSIDE

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ABO Æ, TYPIS FRENCKELLIANIS.,

18



In dimetienda quantitate dilatationis corporum a calorico ita inprimis experimenta sua instituisse invenimus Physicos antiquiores, ut illam pro majore caloris differentia determinarent, plerumque supponentes, cuivis gradui Thermometri alicujus, temperaturam quandam intermediam indicantis, proportionalem esse dilatationem. Maximam scilicet, quam assequi possent, inæqualitatem temperaturarum in experimentis quæsierunt, ut expansiones majores invenirent, atque ut errores, qui evitari nequirent, minimi essent momenti. Hinc accidit, ut adhuc paucorum corporum certe cognoscamus dilatationem pro quovis gradu caloris in Thermometro dato. Exceptis enim legibus expansioris aëris atmosphærici aliarumque aëris specierum, vaporum aquæ puræ & spiritus vini ^{a)}, nec non dilatationis vitri ^{co)}, nullam inven-

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^{a)} Leges hæ expansionis inveniuntur in *Neue Architektura hydraulika von Hrn. PRONY, aus dem Franz. von KARL CHRISTIAN LANGSDORF, Frankf. am Main 1801, zweiter Theil, S. 136 &c.*

^{co)} Vide: Disfert, *de interpolatione pro determinanda vitri*

venimus in scriptis Physicorum formulam, cujus ope aliorum corporum expansionem in data aliqua temperatura determinare possemus. Sic quoque ad determinandum volumen aquæ puræ in diversis temperaturis omni caremus regula certa, etiamsi cognitio hæc utilis omnino esset. Secundum observationes quidem D:ni NOLLET contendunt Physici, volumina aquæ congelantis & ebullientis esse in proportionem $1 : 1,04^{\circ}$); inde autem eo minus de gradu dilatationis in temperaturis aliis mediis concludere licet, quo certius ex recentiorum experimentis apparere videatur, expansionem aquæ pro gradibus puncto congelationis propioribus minorem, pro gradibus vero prope ebullitionis punctum sitis majorem esse.

Qui præterea Thermometris aqueis experimenta de dilatatione aquæ instituerunt, nec non qui gravitatem aquæ specificam corpora solida in illa ponderando determinarunt, contendunt volumen aquæ circa temperaturam $+ 5^{\circ}$ in Thermometro CELSII minimum esse, quibus tamen alii jure objecerunt, considerationem dilatationis instrumentorum suorum illos

dilatatione a calórico, Præfide G. G. HÄLLSTRÖM & Resp. PETRO. CHRIST. SNELLMAN, edit. Abœ 1801, pag. 8. & 9.

*) Vide: PRONY l. c. I Th. 280 f.

los neglexisse, adeoque nihil certi ex eorum experimentis concludi posse *).

Cum inventam habeamus regulam, cujus ope determinari queat pro data caloris temperatura vitri dilatatio **), cumque etiam computare possimus, qualem effectum dilatatio hæc instrumentorum vitreorum in experimentis circa aquæ expansionem factis exferat ***); veram hujus expansionis legem, de qua in temperatura congelationis calori propiore præcipue inter Physicos disputatur, jam inveniri posse contendimus, & quidem nostra de determinanda dilatatione intra puncta 0° & + 20° in Thermometro centesimali conamina paginis sequentibus venia lectoris candidi publici juris facimus.

Satis quidem fuisset commodum, experimenta
in hac disquisitione necessaria instituere cum aqua in-
A 2 clu-

*) Cfr. Disfert. *de methodis inveniendi dilatationes liquorum a calorico*, Præf. G. G. HÄLLSTRÖM & Resp. L. PH. PALANDER, edit. Aboæ 1801, p. 2. fqq.

**) In Disfert. *de interpolatione pro determinanda vitri dilatatione a calorico*, Aboæ 1801.

***). Vid. Disfert. *de methodis inveniendi dilatationes liquorum a calorico*, Aboæ 1801.

elusa in tubo, qui globo instructus esset; cum autem metueremus, ne evaporatio aquæ, cujus mensuram facere non possemus, dimensiones altitudinis in tubo erroneas redderet, cumque etiam determinationem radiorum tubi & globi satis exactam, quæ in calculo necessaria fuisset, instituire nequiremus; corpus vitreum in aqua ponderando hujus dilatationem in diversis temperaturis invenire conati sumus. Uti sumus bilance hydrostatica, a celebri Angliæ mechanico HURTER affabre facta, cui pilo humano appendimus vitrum album, & hujus pondus in aëre atque in aqua destillata exactissime, ut potuimus, observavimus. Differentia horum ponderum diminutionem ponderis in aqua nobis præbuit, quæ in temperatura congelationis sit $= p$, in calore autem graduum m in Thermometro CELSI $= p'$. Si nulla vitri dilatatio fieret, volumen aquæ, quod in temperatura 0° ponitur $= 1$, in calore m graduum esset $= \frac{p}{p'}$; facto

autem vero aquæ volumine $= y$ in hac temperatura, habetur $y = (1 + \frac{(325 + \frac{2m}{62500000})^3 \cdot p}{p'})$, qua æquatione, postquam inventum erat pondus vitri in aëre $= 91718$, ad inveniendum valorem y in sequente Tabula usi sumus.

Gra-

*) Cfr. Disfert. nuperr. cit, p. 11,

Gradus Therm.	Pondus in aqua.	Dimin. ponder.	$p : p'$	γ	$\gamma - 1$
0	53227	38491	1,0000000	1,0000000	0,0000000
+ 1	53221	38497	0,9998442	0,9998592	
+ 2	53217	38501	0,9997402	0,9997727	
+ 3	53215	38503	0,9996884	0,9997360	
+ 4	53213,5	38504,5	0,9996494	0,9997122	
+ 5	53213	38505	0,9996365	0,9997182	— 0,0002818
+ 6	53213	38505	0,9996365	0,9997324	
+ 7	53214	38504	0,9996624	0,9997764	
+ 8	53215	38503	0,9996884	0,9998210	
+ 9	53216	38502	0,9997144	0,9998620	
+ 10	53218	38500	0,9997662	0,9999314	— 0,0000685
+ 11	53220	38498	0,9998182	1,0000012	
+ 12	53222	38496	0,9998702	1,0000720	
+ 13	53224,5	38493,5	0,9999349	1,0001539	
+ 14	53228	38490	1,0000260	1,0002450	
+ 15	53230	38488	1,0000780	1,0003330	+ 0,0003330
+ 16	53233	38485	1,0001560	1,0004287	
+ 17	53236	38482	1,0002339	1,0005282	
+ 18	53239,5	38478,5	1,0003244	1,0006370	
+ 19	53243	38475	1,0004160	1,0007462	
+ 20	53247	38471	1,0005200	1,0008717	+ 0,0008717

Ex hac tabula apparet, si nulla dilatationis vitri ratio habetur, in temperatura + 5° & + 6° maximam videri aquæ densitatem, sicut illam quoque alii diversis modis invenerunt. Si vero in aucta temperatura vitrum dilatatum consideratur, atque calculus observata hac expansione instituitur, propior quidem tem-

peraturæ congelationis invenitur calor ille, in quo aquæ densitas est maxima, non autem hoc in ipsa congelatione accidit, ut putare videntur D:ni VON ARNIM *) & MONGE **). Nam circa quartum vel intra quartum & quintum caloris gradum minimum esse volumen aquæ apparet. Præterea notabilem in hisce experimentis efficit mutationem dilatatio vitri. Non enim parva, adeoque nec negligenda, est differentia inter $\frac{p}{p'}$ & y , quæ etiam in majore calore major est.

Ut appareat lex expansionis aquæ in diversis temperaturis, assumamus æquationem, quæ pro dato aliquo caloris gradu x dabit dilatationem $y - 1$ voluminis ejus, esse hanc: $y - 1 = Ma^x + N\beta^x + \gamma'$, in qua quantitates constantes M, N, a, β, γ' , ope quantitatum in tabula præcedente datarum determinabuntur. Brevitatis causa ponamus

gradibus Thermometri $x = 0; 5; 10; 15; 20;$
respondere valores $y - 1 = 0; a; b; c; d;$
ut,

*) Vid. *Annalen der Physik*, herausgegeben von LUDW. WILH. GILBERT, Halle 1800, 5 B. 1 St. 65 f.

***) Cfr. *Neue Architectura hydraulica*, von PRONY, 1 Th. 280 f.

ut, substitutis his successive in æquatione generali, habeantur sequentes æquationes:

$$\begin{aligned} 0 &= M + N + \gamma', \\ a &= M\alpha^s + N\beta^s + \gamma', \\ b &= M\alpha^{1^0} + N\beta^{1^0} + \gamma', \\ c &= M\alpha^{1^s} + N\beta^{1^s} + \gamma', \\ d &= M\alpha^{2^0} + N\beta^{2^0} + \gamma'. \end{aligned}$$

Subtrahendo quamcunque harum posteriorem a priore exterminatur γ' , & inveniuntur sequentes:

$$\begin{aligned} -a &= M (1 - \alpha^s) + N (1 - \beta^s) \\ a - b &= M\alpha^s (1 - \alpha^s) + N\beta^s (1 - \beta^s) \\ b - c &= M\alpha^{1^0} (1 - \alpha^s) + N\beta^{1^0} (1 - \beta^s) \\ c - d &= M\alpha^{1^s} (1 - \alpha^s) + N\beta^{1^s} (1 - \beta^s) \end{aligned}$$

Si ulterius ab antecedentibus, in α^s ductis subtrahuntur sequentes, eruantur

$$\begin{aligned} -a\alpha^s - a + b &= N (\alpha^s - \beta^s) (1 - \beta^s); \\ (a - b)\alpha^s - b + c &= N\beta^s (\alpha^s - \beta^s) (1 - \beta^s); \\ (b - c)\alpha^s - c + d &= N\beta^{1^0} (\alpha^s - \beta^s) (1 - \beta^s). \end{aligned}$$

Harum æquationum membra posteriora efficiunt seriem geometricam, cujus exponens est β^s , quare membra priora has dant æquationes:

$$\begin{aligned} (-a\alpha^s - a + b)\beta^s &= (a - b)\alpha^s - b + c; \\ ((a - b)\alpha^s - b + c)\beta^s &= (b - c)\alpha^s - c + d; \end{aligned}$$

unde

unde exterminando β^5 eruitur

$$\frac{(a-b)\alpha^5 - b + c}{-a\alpha^5 - a + b} = \frac{(b-c)\alpha^5 - c + d}{(a-b)\alpha^5 - b + c},$$

& evolutis terminis atque positis

$$\begin{aligned} (a-b)^2 + a(b-c) &= A, \quad (a-b)(b-c) + a(c-d) = B, \\ &\& (b-c)^2 - (a-b)(c-d) = C, \text{ erit} \\ A\alpha^{10} - B\alpha^5 + C &= 0. \end{aligned}$$

Hujus vero æquationis radicem quadratam quærendo invenitur

$$\alpha^5 = \frac{B + \sqrt{B^2 - 4AC}}{2A}; \text{ adeoque}$$

$$\alpha = \left(\frac{B + \sqrt{B^2 - 4AC}}{2A} \right)^{1:5}.$$

Inventa sic α , determinatur β^5 ope æquationis

$$\beta^5 = \frac{(a-b)\alpha^5 - b + c}{-a\alpha^5 - a + b} = \frac{B - \sqrt{B^2 - 4AC}}{2A};$$

unde innotescunt

$$\beta = \left(\frac{(a-b)\alpha^5 - b + c}{-a\alpha^5 - a + b} \right)^{1:5} = \left(\frac{B - \sqrt{B^2 - 4AC}}{2A} \right)^{1:5};$$

$$N = \frac{-a\alpha^5 - a + b}{(\alpha^5 - \beta^5)(1 - \beta^5)};$$

$$M =$$

$$M = \frac{-a - N(1 - \beta^5)}{1 - \alpha^5} = \frac{a\beta^5 + a - b}{(\alpha^5 - \beta^5)(1 - \alpha^5)}$$

$$\& \gamma' = -M - N.$$

Si jam substituuntur valores experimentis inventi, scilicet

$$a = 0,0002818; \quad b = 0,0000685;$$

$$c = 0,0003330; \quad d = 0,0008717;$$

inveniuntur

$$M = 0,001008357. \quad \text{Log. } M = 0,0036145 - 3.$$

$$N = 0,000715207. \quad \text{Log. } N = 0,8544320 - 4.$$

$$\alpha = 1,04835314. \quad \text{Log. } \alpha = 0,0205076.$$

$$\beta = 0,74566831. \quad \text{Log. } \beta = 0,8725456 - 1.$$

$$1 + \gamma' = \gamma = 0,9982765.$$

His itaque adhibitis valoribus habetur pro aquæ volumine y in temperatura x graduum, in Thermometro centesimali intra 0 & $+ 20^\circ$ observatorum, determinando hæc æquatio:

$$y = M\alpha^x + N\beta^x + \gamma.$$

Ope hujus æquationis sequentem computavimus tabulam:



Gradus Therm.	Volumen a- que calculo inventum.	Differentia inter vo- lumina quæ experi- mentis & calculo inveniuntur.
0	1,0000000	0,0000000
+ 1	0,9998669	— 0,0000077
+ 2	0,9997824	— 0,0000097
+ 3	0,9997349	+ 0,0000012
+ 4	0,9997156	— 0,0000034
+ 5	0,9997182	0,0000000
+ 6	0,9997381	— 0,0000056
+ 7	0,9997715	+ 0,0000049
+ 8	0,9998161	+ 0,0000049
+ 9	0,9998698	— 0,0000078
+ 10	0,9999314	0,0000000
+ 11	0,9999995	+ 0,0000013
+ 12	1,0000747	— 0,0000027
+ 13	1,0001553	— 0,0000014
+ 14	1,0002413	+ 0,0000037
+ 15	1,0003328	+ 0,0000002
+ 16	1,0004295	— 0,0000008
+ 17	1,0005317	— 0,0000035
+ 18	1,0006392	— 0,0000022
+ 19	1,0007524	— 0,0000062
+ 20	1,0008713	— 0,0000004

Optime itaque experimentis convenire videtur
 calculus ope æquationis $y = Ma^x + N\beta^x + \gamma$ institu-
 tus, & quidem exactissime, si quinque primas notas
 decimales consideraveris; parte enim voluminis cen-
 ties

ties millesima minor obvenit differentia inter volumina, quæ experimentis & calculo inveniuntur.

Ut exacta inveniatur temperatura x , in qua volumen aquæ y minimum evadat, differentietur æquatio $y = Ma^x + N\beta^x + \gamma$, ut, denotante L Logarithmum hyperbolicum, habeatur $dy = Ma^x dx \cdot La + N\beta^x dx \cdot L\beta$, adeoque pro casu minimi y , $\frac{dy}{dx} = Ma^x \cdot La + N\beta^x \cdot L\beta = 0$. Hinc erit $Ma^x La = -N\beta^x L\beta$, atque

$\left(\frac{\alpha}{\beta}\right)^x = -\frac{NL\beta}{ML\alpha} = -\frac{Nl\beta}{Ml\alpha}$, denotante l Logarithmum vulgarem; adeoque invenitur

$$x(l\alpha - l\beta) = lN - lM + l(-l\beta) - l(l\alpha),$$

$$\& x = \frac{lN - lM + l(-l\beta) - l(l\alpha)}{l\alpha - l\beta}.$$

Cumque sit $-l\beta$ quantitas positiva, ut etiam omnes reliquæ quantitates affirmativæ, erit quoque x realis. Est autem in casu præfenti

$$= l\beta$$

$$-l\beta = 0,1274544, \text{ adeoque } l(-l\beta) = -0,8946452$$

$$l\alpha = 0,0205076$$

$$-l(l\alpha) = +1,6880852$$

$$l\alpha - l\beta = 0,1479620$$

$$+0,7934400$$

$$-lM = +2,9963855$$

$$+3,7898255$$

$$lN = -3,1455680$$

$$+0,6442575$$

$$l. 0,6442575 = 0,8090594 - l.$$

$$l. 0,1479620 = 0,1701443 - l.$$

$$lx = 0,6389151.$$

$$\text{adeoque } x = +4,35427;$$

in qua temperatura minimum est aquæ volumen y .

Ut vero hoc volumen innotescat, fiat valoris 4,35427

substitutio pro x in æquatione $y = M\alpha^x + N\beta^x + \gamma$,

quo facto invenitur minimum volumen

$$y = 0,9997143.$$

